Physical Interpretation of the Transparent Absorbing Boundary for the Truncation of the Computational Domain

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***Abstract*— Physical interpretation of the recently introduced transparent absorbing boundary for the grid termination in finite methods shows that the absorbing domain in that method is described by the constitutive equations of moving media. The field solutions in this domain give the requirements for the material parameters to provide zero reflection at arbitrary incidence angles and arbitrary polarizations of the incident waves.**

***Index Terms*—Absorbing boundary, moving media, reflection, truncation method, wave impedance.**

I. Introduction

RECENTLY, a new technique for truncation of the computational domains in finite methods was proposed [1]. In this method, which is called by the authors transparent absorbing boundary (TAB), a physical problem to be solved numerically is transformed into a problem for auxiliary fields. These fields are subject to a set of differential equations which are different from the Maxwell equations, thus this auxiliary problem is treated as a pure mathematical formulation with no clear physical meaning. Here, we demonstrate that this auxiliary problem has a physical counterpart: the equations which govern the auxiliary fields in the absorbing region of TAB are known in physics. They are the equations for the electromagnetic fields in moving media [2], [3]. In other words, the absorbing domain can be treated as if it were filled by a certain uniaxial bianisotropic absorbing material. We will show that the absorption is caused solely by the field coupling, not by dielectric or magnetic losses. Field solutions of the equivalent physical problem lead to the requirements necessary to provide zero reflection from the computational domain boundary.

II. Theory

A. Physical Interpretation

In the TAB boundary formulation, auxiliary vectors $\boldsymbol{E}$ and $\boldsymbol{H}$ are introduced, which are related to the physical fields $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ by [1, eq. (1)]

$$\boldsymbol{E}(t,r) = F(r)\boldsymbol{E}_0(t,r), \quad \boldsymbol{H}(t,r) = F(r)\boldsymbol{H}_0(t,r). \tag{1}$$

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Theoretically, ideal absorption takes place if the function $F(r)$ decays outward from the computation domain and becomes zero at the truncation boundary. Instead of the Maxwell equations for the physical fields $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$, equations for the auxiliary fields are solved. These equations ([1, eq. 2]) can be rewritten for source-free regions (in the frequency domain) in the form

$$\nabla \times \boldsymbol{H} = j\omega\epsilon_0\epsilon\boldsymbol{E} + \frac{1}{F}\nabla F \times \boldsymbol{H} \tag{2}$$

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0\mu\boldsymbol{H} + \frac{1}{F}\nabla F \times \boldsymbol{E}. \tag{3}$$

Here, the relative complex permittivity and permeability ϵ and μ incorporate the loss parameters σ and σ^*, explicitly included in [1, eq. (2)]. Clearly, these equations can be interpreted as conventional macroscopic Maxwell equations in the medium with the following constitutive relations:

$$\boldsymbol{D} = \epsilon_0\epsilon\boldsymbol{E} - \frac{j}{\omega F}\nabla F \times \boldsymbol{H} \tag{4}$$

$$\boldsymbol{B} = \mu_0\mu\boldsymbol{H} + \frac{j}{\omega F}\nabla F \times \boldsymbol{E}. \tag{5}$$

Equations (2c) and (2d) of [1] can be seen to coincide (in source-free regions) with $\nabla \cdot \boldsymbol{D} = \nabla \cdot \boldsymbol{B} = 0$. Function F can be chosen to be a linearly decaying function, thus $-\nabla F$ is a constant vector pointing outwards from the computational domain. With this in mind, we can consider the case when the auxiliary fields $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy conventional Maxwell equations in a homogeneous region whose material properties are described by constitutive relations (4) and (5).

Material relations (4) and (5) (see [6, p. 91]) describe a lossy nonreciprocal (see [6, p. 94]) uniaxial bianisotropic material [5], more specifically, a moving medium whose velocity is along the direction of ∇F [2], [3].

B. Plane-Wave Solutions and Discussion

Knowledge of plane-wave solutions in the absorbing region helps in understanding its properties and gives the zero-reflection requirement. With the goal to find the eigensolutions for the Maxwell equations with the constitutive relations of TAB (4), (5) we note that this problem is similar to that for the uniaxial omega medium (composite spatially dispersive material with inclusions in the form of letter Ω). The only difference is that the omega medium is reciprocal, thus both the field-coupling terms in its constitutive relations have the same sign. Wave solutions for uniaxial omega media are known [7], and the present problem can be solved in a similar manner. Let the direction of ∇F is defined by the unit vector $\boldsymbol{z}_0$. The following notation is convenient for the field analysis: $\nabla F/(\omega F) = \sqrt{\epsilon_0\mu_0} V \boldsymbol{z}_0$. Here, the field coupling coefficient V (proportional to the velocity of the moving medium) is dimensionless.

To find the eigensolutions we split the fields into the transverse and longitudinal parts with respect to the axis z: $\boldsymbol{E} = \boldsymbol{E}_t + \boldsymbol{z}_0 E_z$, $\boldsymbol{H} = \boldsymbol{H}_t + \boldsymbol{z}_0 H_z$. Considering eigensolutions, the coordinate dependence of the fields in the transverse plane can be substituted in the form $\exp(-j\boldsymbol{k}_t \cdot \boldsymbol{r})$, where $\boldsymbol{k}_t$ is a two-dimensional vector in the transverse plane. The following equations for the transverse field components hold:

$$\left(\frac{d}{dz} - k_0 V\right)\boldsymbol{E}_t = \left(j\omega\mu_0\mu\overline{\overline{I}}_t - \frac{j}{\omega\epsilon_0\epsilon}\boldsymbol{k}_t\boldsymbol{k}_t\right) \cdot \boldsymbol{z}_0 \times \boldsymbol{H}_t \tag{6}$$

$$\left(\frac{d}{dz} - k_0 V\right)\boldsymbol{z}_0 \times \boldsymbol{H}_t = \left(j\omega\epsilon_0\epsilon\overline{\overline{I}}_t - \frac{j}{\omega\mu_0\mu}\boldsymbol{z}_0 \times \boldsymbol{k}_t\boldsymbol{z}_0 \times \boldsymbol{k}_t\right) \cdot \boldsymbol{E}_t \tag{7}$$

where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the free-space wave number and $\overline{\overline{I}}_t = \overline{\overline{I}} - \boldsymbol{z}_0\boldsymbol{z}_0$ is the transverse unit dyadic. Eigensolutions of (6) and (7) are plane waves modulated by $F(z)$

$$\boldsymbol{E}_t = F(z)(\boldsymbol{A}\exp(j\sqrt{k_0^2\epsilon\mu - k_t^2}) + \boldsymbol{B}\exp(-j\sqrt{k_0^2\epsilon\mu - k_t^2})) \tag{8}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are transverse amplitude vectors. This result can be expected: the auxiliary fields are products of the physical fields (which are plane-wave solutions of the Maxwell equations) and function F. Thus, the fields indeed always decay in the outward direction, whatever the propagation direction of a wave. This property is a manifestation of the nonreciprocity of the effective medium. The decay factor is defined by the additional parameter in the constitutive relations V.

Substituting the eigensolutions in (6), we find the wave impedances of the eigenwaves. In fact, parameter V cancels out, and the impedances are seen to coincide with that of a simple isotropic medium with the same permittivity and permeability:

$$Z_{\mathrm{TM}} = \eta\sqrt{1 - k_t^2/(k_0^2\epsilon\mu)}, \quad Z_{\mathrm{TE}} = \frac{\eta}{\sqrt{1 - k_t^2/(k_0^2\epsilon\mu)}} \tag{9}$$

where $\eta = \sqrt{\mu/\epsilon}$. Indexes TM and TE refer, respectively, to TM and TE polarizations with respect to the axis z.

This result leads to the conclusion that the ideal interface matching with free space can be realized if $\epsilon = \mu = 1$ since in that case the wave impedances in the absorbing region are the same as that of free space. If the computational domain is filled with a certain isotropic material, the permittivity and permeability in the absorbing region must be the same as that in the main region. The theory was developed in the frequency domain, and the permittivity and permeability in (9) are complex numbers. In the time domain, this requirement concerns real-valued permittivity, permeability, and also conductivities.

Absorption in the case of matching with lossless media is solely due to the coupling parameter V. The decay rate is independent from the permittivity and permeability values. Since the wave impedances (9) are the same as that of a simple magnetodielectric, the reflection from an interface does not depend on function F.

III. Conclusion

The recently introduced TAB for the termination of computational domain of finite methods is seen to be equivalent to a material layer described by the same constitutive equations as hold in moving media. This “filling material” is a lossy nonreciprocal uniaxial bianisotropic medium. Electric and magnetic fields are coupled, and the coupling parameter is determined by the function which connects physical and auxiliary fields in TAB. On the other hand, the permittivity and permeability of the absorbing layer do not depend on that function.

Field solutions for regions filled by this material show that eigenwaves in TAB are ordinary plane waves modulated by function $F(z)$. Field decay is totally due to the artificially introduced field coupling and does not depend on the frequency. Wave impedances for eigenwaves in the absorbing regions are seen to be the same as that in the computational domain (which means that no reflection at the interface occurs for all incidence angles and arbitrary polarizations) only if the permittivity, permeability, and conductivities in the absorbing region are the same as that in the main computational domain. In fact, ideal matching requirement is the same as that for interfaces with simple isotropic media.

The above field analysis of TAB can be easily extended to the case when the permittivity and permeability are uniaxial dyadics, like in the anisotropic PML formulation [4], [5]. The result shows that the two loss mechanisms, those employed in the PML and the field coupling of TAB, act independently. The zero-reflection condition does not change with the introduction of bianisotropic coupling parameters. The trivial zero-reflection requirement (free-space permittivity and permeability of the layer) can be exploited in TAB because the loss is provided due to the field coupling. Finally, we note that [as seen from (8)] the waves traveling back into the computational domain from the truncation boundary are actually amplified in the same rate as the outward waves are attenuated, which means that the method is very sensitive to reflections from that boundary.

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References

[1] J. Peng and C. A. Balanis, “Transparent absorbing boundary (TAB) for the truncation of the computational domain,” *IEEE Microwave Guided Wave Lett.*, vol. 7, pp. 347–349, Oct. 1997.

[2] C. T. Tai, "A study of electrodynamics of moving media," *Proc. IEEE*, vol. 52, pp. 685–689, June 1964.
[3] H. C. Chen and D. K. Cheng, "Constitutive relations for a moving anisotropic medium," *Proc. IEEE*, vol. 54, no. 1, pp. 62–63, 1996.
[4] Z. S. Sacks, D. M. Kingsland, R. Lee, and J.-F. Lee, "A perfectly matched anisotropic absorber for the use as an absorbing boundary condition," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1460–1463, 1995.
[5] S. D. Gedney, "An anisotropic perfectly matched layer—Absorbing medium for the truncation of FDTD lattices," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1630–1639, Dec. 1996.
[6] S. A. Tretyakov, A. H. Sihvola, A. A. Sochava, and C. R. Simovski, "Magnetoelectric interactions in bi-anisotropic media," *J. Electromagn. Waves Applicat.*, vol. 12, pp. 481–497, 1998.
[7] S. A. Tretyakov and A. A. Sochava, "Novel uniaxial bianisotropic materials: Reflection and transmission in planar structures," in *Progress in Electromagnetics Research*, vol. PIER9, pp. 157–179, 1994.
[8] I. V. Lindell, *Methods for Electromagnetic Field Analysis*. Oxford, U.K.: Clarendon, 1992.